Analysis and Realization of a Pulsewidth Modulator Based on Voltage Space Vectors

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Abstract—The method of using a space vector concept for deriving the switching instants for pulsewidth-modulated voltage source inverters is compared with the commonly used established sinusoidal concept. After deducing the switching times from assumptions for minimum current distortion, the resulting mean voltage values are shown and the differences between these and the established sinusoidal PWM are elaborated. Based on an analytical calculation, the current distortions and torque ripples are evaluated and compared with those of established sinusoidal pulsewidth modulation. The realization of a pulsewidth modulator is described in the range of synchronous sinusoidal pulsewidth modulation, which generates—by software—symmetrical synchronous pulse patterns. The performance has been proved by laboratory tests.

INTRODUCTION

VARIABLE-FREQUENCY ac drives are increasingly used for various applications in industry and traction. Due to the improvement of fast-switching semiconductor power devices, voltage source inverters with pulsewidth-modulated (PWM) control, as shown in Fig. 1, find particularly growing interest.

Control methods that generate the necessary PWM patterns have been discussed extensively in the literature. Two basic concepts may be distinguished: for small and medium-size drives, the current-controlled PWM has proved to be advantageous. For big drives employing inverters with low switching frequency, PWM voltage control is more advantageous. However, this method is also an option for small and medium-size drives. A detailed description of the basic methods may be found in [1] and [2]. This paper deals with PWM voltage control methods.

PWM Voltage Control Methods

If the switching frequency $f_s$ is high compared to the fundamental output frequency $f_1$ of the inverter (frequency ratio $z_T \geq 9, \cdots, 15$), sampling methods with sinusoidal reference signals, e.g., near sampling [3] or regular sampling [4], may be used. These methods aim at generating an inverter output voltage with sinusoidally modulated pulse-widths. They will be referred to as established sinusoidal modulation in this paper. If the switching frequency is an integer multiple of the fundamental output frequency, this will be called synchronized sinusoidal modulation, in contrast to free-running sinusoidal modulation, where the switching frequency and the fundamental output frequency are independent of each other.

Sinusoidal modulation maintains good performance of the drive in the entire range of operation between zero and 100-percent modulation index $m$. If the modulation index exceeds unity, sinusoidal modulation is no longer possible. In this range of operation it is useful to apply optimized PWM with precalculated switching points, which are stored in a look-up table. Optimized PWM is also recommended for low-frequency ratios. It is practical to calculate the notches off-line in such a way that current ripple or losses or torque pulsations are minimized.

Sinusoidal Modulation

In the following the sinusoidal modulation will be discussed in more detail. For this purpose an inverter leg, as shown in Fig. 2(a), is represented by an equivalent circuit, shown in Fig. 2(b).

If the changeover switch is operated according to sinusoidal modulation, the line-to-center voltage $U_l$ follows a shape as shown for instance in Fig. 3 in per-unit notation. The mean value of the voltage $u$ averaged over each cycle of the switching frequency follows a staircase shape, which will be replaced with a smoothed curve $u(t)$ for sufficiently high frequency ratios $z_T$. This curve will be called mean value herein. In a symmetrical three-phase inverter the mean line-to-center voltages are

$$\bar{u}_l(t) = \bar{U}_l(t) = m \cdot \sin \omega_1 t = \bar{u}_2 \left(1 - \frac{T}{3}\right) = \bar{u}_3 \left(1 + \frac{T}{3}\right).$$

(1)
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Fig. 2. (a) Inverter leg. (b) Inverter leg equivalent circuit.

Fig. 3. Single-phase inverter output voltage by established sinusoidal modulation.

However, the instantaneous values of the voltages depend on the method of pulsing. It has been shown that a symmetry of the pulses within one switching period, as shown in Fig. 7, is particularly advantageous [1], [11]. For fixed-frequency ratios that are multiples of three, this concept results in equal shapes of the three phase-to-center voltages:

\[ u_1(t) = u_2 \left( t - \frac{T}{3} \right) = u_3 \left( t + \frac{T}{3} \right) \] (2)

\[ z_f = \frac{f_1}{f_2} = 3 \cdot n \quad (n = 1, 2, 3, \ldots). \] (3)

The upper limit of the phase-to-center voltage for sinusoidal modulation is theoretically \( m = 1 \) with \( \bar{u} = 1 \). This is only 78 percent of the value that would be reached by square-wave operation.

**SPACE-VECTOR MODULATION**

A different approach to PWM modulation is based on the space vector representation of the voltages in the \( \alpha, \beta \) plane. The \( \alpha, \beta \) components are found by Park transform, where the total power, as well as the impedances, remains unchanged (see also (23)). This concept was discussed in publications such as [4]-[6]. For our purpose the basic ideas are summarized.

The three machine voltages are represented by a voltage space vector \( \vec{U} \). There are eight states available for this vector according to eight switching positions of the inverter, which are depicted in Fig. 4.

We define a mean space vector \( \overline{\vec{U}} \), which is almost constant during a switching period. This vector generates the fundamental behavior of the machine, e.g., currents and torque. The difference space vector \( \vec{U} - \overline{\vec{U}} \) causes the current harmonics. For discussing the current harmonics, a simplified equivalent circuit of the induction machine according to Fig. 5(c) can be used.

For the quality of PWM control, the deviation of the machine currents from the fundamental currents is highly important. For the ideal case of sufficiently high frequency ratios, cycles of switching states can be found wherein the mean voltage vector is equalized to the fundamental voltage vector and the average deviation current vector becomes zero.

Within one switching state the deviation current vector changes by the amount

\[ \Delta \vec{T} = \frac{1}{L \omega} \int_{t_1}^{t_2} (\vec{U} - \overline{\vec{U}}) \, dt \quad \text{min} \] (4)

where

- \( t_1 \): beginning of switching state.
- \( t_2 \): end of switching state.

An optimum PWM modulation is expected if
- the maximum deviation of the current vector for several switching states becomes as small as possible, and
- the cycle time is as short as possible.

Fig. 4. (a) Inverter switching states. (b) Inverter output voltage space vectors.

Fig. 5. Equivalent circuit of induction machine. (a) Single-phase representation. (b) Simplification for harmonic current calculation. (c) Simplification for space vector calculation.
These conditions are met in general if
- only the three switching states adjacent to the reference vector are used, and
- the cycle wherein the average voltage vector becomes equal to the reference vector consists of three successive switching states only.

To obtain minimum switching frequency of each inverter leg it is necessary to arrange the switching sequence in such a way that the transition from one state to the next is performed by switching only one inverter leg.

It can be seen in the switching diagram of Fig. 4 that these conditions are met if starting from one zero state the inverter legs are switched in the relevant sequence ending at the other zero state. If, for instance, the reference vector sits in sector I, the switching sequence has to be \( \cdots 812721812721 \cdots \).

Hence it follows for a switching cycle in sector I:

\[
\int_{0}^{T_z} U_{\text{ref}} dt = \int_{0}^{T_z} U_{\text{d}} dt = \int_{0}^{T_z} U_{\text{d}} dt = \int_{0}^{T_z+T_1} U_{\text{d}} dt + \int_{0}^{T_z+T_2} U_{\text{d}} dt. \tag{5}
\]

For sufficiently high switching frequency the reference space vector \( U_{\text{ref}} \) is assumed constant during one switching cycle.

Taking into account that the states \( U_{\text{1}} \) and \( U_{\text{2}} \) are constant and \( U_{\text{d}} = 0 \), one finds (see Fig. 6)

\[
U_{\text{1}} \cdot T_1 + U_{\text{2}} \cdot T_2 = U_{\text{ref}} \cdot T_z. \tag{6}
\]

If the space vectors in this equation are described in rectangular coordinates, it follows that

\[
T_1 \cdot \sqrt{3} U_{\text{d}} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + T_2 \cdot \sqrt{3} U_{\text{d}} \cdot \begin{bmatrix} \cos 60^\circ \\ \sin 60^\circ \end{bmatrix} = T_z \cdot \sqrt{3} U_{\text{d}} \cdot a \cdot \begin{bmatrix} \cos \gamma \\ \sin \gamma \end{bmatrix} \tag{7}
\]

\[
0 \leq \gamma \leq 60^\circ, \quad a = \frac{|U_{\text{ref}}|}{\sqrt{3} U_{\text{d}}}. \tag{8}
\]

Hence

\[
T_1 = T_z \cdot a \cdot \frac{\sin (60^\circ - \gamma)}{\sin 60^\circ} \tag{9}
\]

\[
T_2 = T_z \cdot a \cdot \frac{\sin \gamma}{\sin 60^\circ} \tag{10}
\]

\[
T_3 = T_3 = T_0 = T_2 - T_2 - T_1. \tag{11}
\]

For the sectors II–VI the same rules apply. This results in a definite switching order according to Fig. 7, which shows exactly the same symmetry as derived for established sinusoidal modulation [11].

To compare the results of the space-vector PWM with other PWM concepts, the mean values of the phase-to-center voltages averaged over one switching cycle are evaluated. For sector I one finds

\[
U_i(t) = U_q \left( -\frac{T_0}{2} + T_1 + T_2 + \frac{T_0}{2} \right) = \frac{2}{\sqrt{3}} a \cdot U_q \cdot \sin (\gamma + 60^\circ) \tag{12}
\]

\[
U_2 = U_q \left( -\frac{T_0}{2} - T_1 + T_2 + \frac{T_0}{2} \right) = 2a \cdot U_q \cdot \sin (\gamma - 30^\circ) \tag{13}
\]

\[
U_3 = U_q \left( -\frac{T_0}{2} + T_2 - T_2 + \frac{T_0}{2} \right) = -U_1. \tag{14}
\]

Taking into account the necessary changes in the other sectors, one finds for the fundamental period

\[
t(t) = a \cdot U_q \tag{15}
\]

\[
U_i(t) = \begin{cases} 
2 \sin \omega t & \text{for } 0 \leq \omega t \leq 30^\circ \\
\frac{2}{\sqrt{3}} \sin (30^\circ + \omega t) & \text{for } 30^\circ \leq \omega t \leq 90^\circ
\end{cases} \tag{16}
\]

\[
U_i(t) = U_i(-t) = -U_i(t-T/2) \tag{16}
\]

\[
U_i(t) = U_i(t-T/3) = U_i(t+T/3). \tag{17}
\]
Hence the phase-to-phase voltages are

\[
U_{12}(t) = U_1(t) - U_2(t) = \frac{4}{\sqrt{3}} a \cdot U_q \cdot \sin (\omega_1 t + 30^\circ) \tag{18}
\]

\[
U_{32}(t) = U_{32}(t - T/3) = U_{32}(t + T/3). \tag{19}
\]

Both the phase-to-center voltage \( \dot{U}_1 \) and the phase-to-phase voltage \( \dot{U}_{12} \) are shown in Fig. 8 in per-unit notation. It turns out that with space-vector PWM, the phase-to-phase voltage which is seen by the machine is sinusoidal, as expected. However, the phase-to-center voltage is not sinusoidal. To understand this result we have to recall that the Park transformation, which defines the components of the space vectors, is not definitely reversible. Harmonics of triplen orders may be added to the phase voltages without affecting the Park components and the value of the phase-to-phase voltages. Once this curve is known it could be substituted for the sinusoidal reference signal in a normal three-phase modulator. In fact an infinite number of curves could be synthesized as reference curves for PWM modulators. Some have been mentioned in the literature, e.g. [8], [9].

Comparing the phase-to-phase voltages according to the established PWM and the space-vector PWM, one finds

\[
m = \frac{4}{3} \cdot a. \tag{20}
\]

Looking at Fig. 4 one finds that the maximum value for a sinusoidal PWM is \( a = \sqrt{3}/2 \).

This leads to the maximum modulation index in the case of space vector PWM:

\[
m_{\text{max}} = 2/\sqrt{3}. \tag{21}
\]

This is approximately 15 percent more than with established PWM [8], [9]. The amplitude of the maximum voltage fundamental reaches about 90 percent of the respective square-wave fundamental.

**Comparison of Sinusoidal PWM and Space Vector PWM**

It has been found in the previous section that with space-vector modulation a modulation index of \( m = 1.15 \) can be reached without any constraint, whereas with sinusoidal modulation, notches are suppressed and low-order harmonics occur in the range of overmodulation \( m > 1 \). In addition, the harmonic content of the inverter output voltages and currents is less for the space-vector method than for its counterpart. This is demonstrated in Fig. 9, which shows the results of simulation runs with exact modeling of the modulation procedures. In either case the same frequency ratio \( z_\tau = 21 \) and the same modulation index \( m = 0.9 \) have been used. Because the phase-to-phase voltage, which has the shape of \( u_{ab} \), is more favorable for the space-vector PWM, the harmonic content of the current and the torque pulsations are significantly reduced. To demonstrate the differences between the two modulation concepts, an analytical solution is used also for the sinusoidal PWM [10], [11].

Under the same assumptions as those described for the space-vector PWM, a single-phase approach is used as a first step. On the basis of an equivalent circuit for the induction machine, as shown in Fig. 5(b), the phase-to-center voltage, the mean phase-to-center voltage, and the harmonic content of the current are shown in Fig. 10.

In analytical form the harmonic content of the current is

\[
I_{\alpha\mu}(t) = \frac{U_d}{L_a} \cdot \begin{cases}
(1 + \alpha_\mu) t & \text{for } 0 \leq t \leq \frac{1 + \alpha_\mu}{4f_s} \\
(1 + \frac{\alpha_\mu}{2f_s}) - (1 - \alpha_\mu) t & \text{for } \frac{1 + \alpha_\mu}{4f_s} < t \leq \frac{1}{2f_s}
\end{cases} \tag{22}
\]

\( \mu = 1, 2, 3 \).

For the symmetrical three-phase machine with open neutral the harmonic content of the phase currents can be derived from the line-to-line voltages. Under the symmetry conditions that we have supposed both for established sinusoidal modulation and space vector modulation (refer to Fig. 7), the Park components of the harmonic content of the stator currents can be derived by superimposing the single-phase values (22) [10], [11]:

\[
\begin{bmatrix}
I_{1a} \\
I_{1b} \\
I_{1c}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 - 1/2 & -1/2 & 1 \\
-1/2 & 1 & -1/2 \\
-1/2 & -1/2 & 1
\end{bmatrix} \begin{bmatrix}
I_{11} \\
I_{12} \\
I_{13}
\end{bmatrix}. \tag{23}
\]

The harmonic content of the stator currents can be calculated in a similar way:

\[
\begin{bmatrix}
I_{a} \\
I_{b} \\
I_{c}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -1/2 & -1/2 \\
-1/2 & 1 & -1/2 \\
-1/2 & -1/2 & 1
\end{bmatrix} \begin{bmatrix}
I_{11} \\
I_{12} \\
I_{13}
\end{bmatrix}. \tag{24}
\]

To compare the copper losses, the rms value of the harmonic machine current is to be evaluated. Since the harmonic content is orthogonal to the fundamental current, the additional copper losses due to current harmonics can be calculated independently. By integration of (23) and (24) one finds for sinusoidal PWM

\[
P_{a} = \frac{2}{3} P_{c} = \frac{2}{3} \left( \frac{U_d}{f_5 L_a} \right)^2 \cdot \frac{3}{2} \frac{m^2 - 4 \cdot \sqrt{3}}{\pi} \cdot \frac{m^3}{3} + \frac{9}{8} \frac{m^4}{48}. \tag{25}
\]
Fig. 9. Simulation of PWM inverter-fed induction machine. (a) Established sinusoidal PWM \( f_s/f_1 = 21; m = 0.9 \). (b) Space-vector PWM \( f_s/f_1 = 21; m = 0.9 \).

(a) Established sinusoidal PWM (b) Space-vector PWM

Fig. 10. Voltage pulse and resulting harmonic current within one pulse period for single-phase system.

A similar calculation for the space-vector concept yields

\[
F_{A}^2 = \left( \frac{U_q}{f_s L_s} \right)^2 \cdot \frac{3}{2} \cdot \frac{m^2}{\pi} - \frac{4\sqrt{3}}{8} \cdot \frac{9}{2} \cdot \frac{9\sqrt{3}}{8\pi} \cdot m^4
\]

Fig. 11 shows the evaluation of these equations in per-unit notation with \( F_{A}^2 = F_{A}^2 \cdot (f_s L_s / U_q)^2 \).

For cross-reference to the fundamental quantities the ideal no-load machine current may be used:

\[
I_0 = \frac{m \cdot U_q \cdot \sigma}{\sqrt{2} \cdot 2\pi f_1 \cdot L_s}.
\]

For both PWM methods the envelope curves are plotted in Fig. 12. As expected, a strong dependence on the modulation index \( m \) appears. Then the maxima of these curves are evaluated as a function of the modulation index. These values are given in analytical form for sinusoidal PWM and for

\[
T_{H_m}(\psi) = \max_{0 < \psi < \pi} \left\{ T_{H_m}(\psi) \right\}
\]

(29)

The torque pulsations do not exceed the following limits:

\[
T_{\bar{H}_m} = \max_{0 < \psi < 2\pi} \left\{ T_{H_m}(\psi) \right\}
\]

(28)

where the angle \( \psi \) describes the position within the fundamental period with respect to the zero crossing of \( U_q \). This equation is to be evaluated for both PWM modes. As the relation between \( \psi \) and \( t \) depends on the point of operation and cannot be represented in simple analytical form, an envelope curve will be derived:

\[
T_{H_m}(\psi) = \max_{0 < \psi < \pi} \left\{ T_{H_m}(\psi) \right\}
\]

(28)
space-vector PWM in (31) and (32), respectively:

\[
\hat{T}_p = \frac{\sqrt{6}}{16} (2m - m^2) \cdot \frac{U_p \cdot \dot{\psi}}{f_s \cdot L_s^*} \quad (31)
\]

\[
\hat{T}_p = \frac{\sqrt{6}}{16} (2m - \frac{3}{2} m^2) \cdot \frac{U_p \cdot \dot{\gamma}}{f_s \cdot L_s^*} \quad (32)
\]

In (32) a slight inaccuracy in the range \( m > 1 \) was accepted for the sake of a simple result.

The evaluation of (31) and (32) is depicted in Fig. 13. It is apparent that the space-vector PWM concept results in considerably less torque pulsations than the sinusoidal PWM. Again in Fig. 13 the torque pulsations are plotted in per-unit notation. For comparison with the fundamental quantities the pull-out torque may be used. This is under ideal assumptions:

\[
T_{p,n} = \frac{1}{2} \sqrt{\frac{3}{2}} m \cdot \frac{U_p \cdot \dot{\gamma}}{2\pi f_s L_s^*} \quad (33)
\]

**PWM MODULATOR**

A modulator for practical use in high-power thyristor inverters or GTO inverters should be capable of the following modes of operation: free-running sinusoidal modulation, synchronous sinusoidal modulation, and optimized PWM including overmodulation and square-wave operation. A smooth transition between these modes is highly desirable. A versatile modulator with a general strategy and a high degree of flexibility can be realized by means of a microprocessor.

The real-time forming of the inverter gate pulses, however, is time critical. For this reason this is usually the task of separate counters, which are controlled by the microprocessor.

One problem of this solution is the synchronization of the pulse frequency with the fundamental working frequency, which is necessary for small frequency ratios (about \( z_f \leq 30 \)).

For synchronized pulsing the clock frequency of the counters is

\[
f_{CLK} = 2 \cdot n_c \cdot z_f \cdot f_s \quad (34)
\]

where \( n_c \) is the resolution of the entire pulse cycle. For an adequate resolution of the pulse angle the counter frequency may reach the megahertz range.

If this frequency is derived from a higher reference frequency by dividing, only a limited number of frequency ratios is possible due to the technological upper limit of the reference frequency. If the counter frequency is generated by phase-locked-loop (PLL) circuits, other problems arise due to PLL transients if the frequency ratio is changed, or due to the symmetry requirements of the pulse angles. These problems may be overcome by very complex PLL arrangements using several PLL loops. It seems to be more practical to adjust the switching frequency by means of software. In this case the counters are clocked with fixed stabilized frequency, and the time positions of the notch angles are loaded into the counters by the microprocessor. Therefore the microprocessor has to calculate the count value for the cycle time \( T_c \) from the digital value of the reference fundamental frequency and the suitable frequency ratio:

\[
T_c = \frac{1}{2\pi f_s} \cdot f_i \quad (35)
\]

The cycle time then has to be multiplied with the pulsewidth ratio derived from the reference curve for the PWM (see (9) and (10)).

**DESIGN OF A PWM CONTROLLER**

On the basis of the space vector concept a PWM controller was developed. It covers all modes of operation that were mentioned in the previous section. The reference values of the fundamental output voltage, which for synchronous modulation have to be the amplitude and the frequency, are given in purely digital form. This enables the modulator to be interfaced directly with a superimposed digital drive control system.

As an example the synchronous sinusoidal modulation is depicted in Fig. 14. As described previously the reference curve and the cycle time are calculated by software in a space-vector reference frame. The hardware counters for the regular sampling procedure could be organized either in the space-vector system or in the three-phase system. The authors preferred the three-phase arrangement as depicted in Fig. 14. The sampling instants are fixed for strongly symmetrical pulsing, and a software counter, which is loaded with the number of cycles per sector, selects for every frequency ratio the suitable sampling values in the reference curves. The reference curves, as seen in Fig. 8, are calculated piecewise in two channels and multiplied with the modulation index and the cycle time. The sector distributor selects the respective values for the three phases. As the regular sampling method is used, only one reference value is needed for each switching cycle. A new value of the frequency ratio can be started only at the beginning of a 60° sector. Thus fully symmetric pulse patterns are generated. Since with space-vector PWM the current distortion in the symmetry axis of the pulses is zero (see Fig. 10), this ensures pulse ratio transitions without current and torque transients.

As an example of the performance of this modulation, Fig. 15 shows the oscillogram of voltage and current as measured at a laboratory test bed with no load and a fundamental frequency of about 23 Hz. One sees that the frequency ratio is changed without any current transient.

Operation in another mode is possible by just calling the appropriate subroutine. The fundamental frequency and the
Fig. 14. Principle of controller for synchronous sinusoidal modulation.
modulation index can be changed independently. The transition from one mode to another is without current transients. The described modulator was realized by means of an 8086 microprocessor. Although rather complex programs are needed, the modulator is capable of switching frequencies up to 2 kHz.

CONCLUSION

It was shown that sinusoidal modulation generated in a space vector representation has the advantages of lower current harmonics and a possible higher modulation index compared with the three-phase sinusoidal modulation method. A modulator on the basis of an 8086 microprocessor suitable of operating in the free-running mode, in the sinusoidal modulation mode, and in the fixed-pattern mode was developed. As an example the operation of this modulator was shown in the sinusoidal modulation mode.

NOMENCLATURE

- \( U_d \) DC source voltage.
- \( U_{dc} \), \( \mu = 1, 2, 3 \) Phase-to-center voltage.
- \( u_{dc}(t) = U_{dc}(t)/U_d \) Per-unit value of \( U_{dc}(t) \).
- \( U_0(f) \) Fundamental amplitude of \( U_{dc}(t) \).
- \( U(t) = f_{dc} \frac{d}{dt} U(t') dt' \) Mean value of \( U(t) \) for switching period \( \tau = 1/f \).
- \( U_{pA}, U_{pB}, U_{pC} \) Park components of voltage.
- \( U_{pA}, U_{pB}, U_{pC} \) Phase-to-phase voltage.
- \( \bar{U} \) Voltage space vector.
- \( I_A, I_B, I_C \) Machine currents (star connection).
- \( I_{pA}, I_{pB}, I_{pC} \) Park components of \( I_A, I_B, I_C \).
- \( I_{pA}(t), \mu = 1, 2, 3 \) Harmonic content of current for single-phase calculation.
- \( P_{hA} = f_{dc} \frac{d}{dt} P_{hA}(t) dt \) RMS value of harmonic content of current.
- \( I_0 \) RMS value of fundamental machine current at no-load.
- \( \phi \) Flux, crest value.
- \( T_\gamma \) Torque pulsation.
- \( T_{H-}(\varphi) \) Envelope curve of torque pulsations.

\[ T_\gamma = \frac{f}{f_p} \]
\[ f_p = \frac{1}{T} \]
\[ f_p = \frac{1}{\tau} = \frac{1}{2(T_0)} \]
\[ z = \frac{U_p(f)}{U_d} \]
\[ m = \frac{f}{f_p} \]
\[ \sigma = \frac{I_p}{I_0} \]

\[ L_a \]
\[ L_c^* \]

\[ \gamma = 2\pi f_1 \left( \frac{2}{3} \right) \]

\[ \frac{2\pi f_1}{3} \leq \varphi \leq 60^\circ \]

\[ \frac{2\pi f_1}{3} \leq \varphi \leq 360^\circ \]

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